Negenproef

Doel: controle van berekening

12	1+2 = 3
× 257	2+5+7 = 14 → 1+4 = 5
=3084	3+0+8+4 = 15 → 1+5 = 6
	3 × 5 = 15 → 1+5 = 6

De wiskunde:

a_i is i^e cijfer van a

$$a = \sum_{i=0} 10^i a_i = \sum_{i=0} (9+1)^i a_i$$

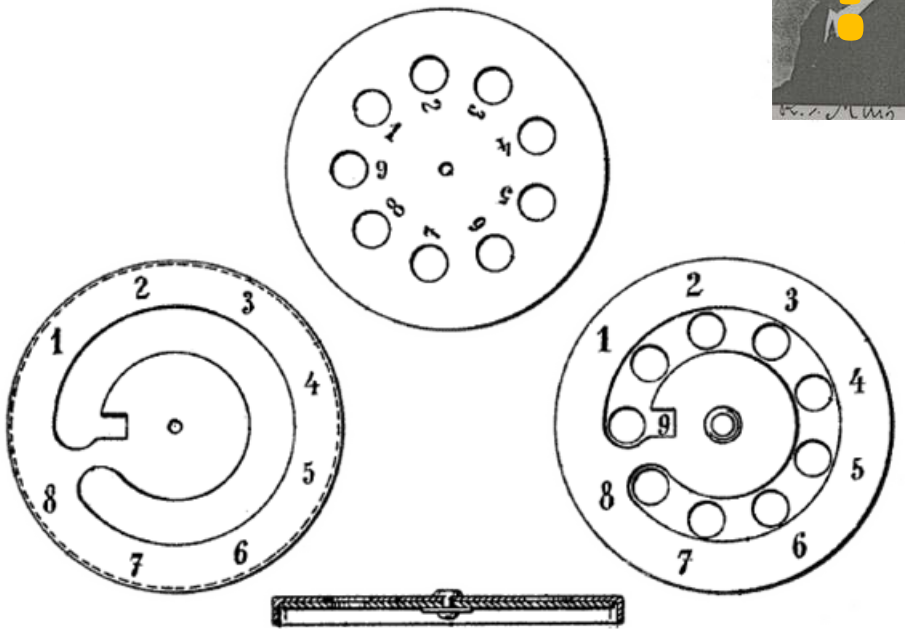
$$a \bmod 9 = \left(\sum_{i=0} a_i\right) \bmod 9$$

$$a \circledast b = c \qquad (a \bmod 9) \circledast (b \bmod 9) = c \bmod 9$$

$\circledast$ is $+$ of $-$ of $\times$ of $\div$

Calculator:

Rudolf Malý, 1914



Elfproef

Doel: controle van berekening

12	−1+2 = 1
× 257	2−5+7 = 4
=3084	−3+0−8+4 = −7 ➡ 11−7 = 4
	1 × 4 = 4

De wiskunde:

$i = 0$: eenheid, 2: honderdtal, … | $i = 1$: tiental, 3: duizendtal, …

$$a \bmod 11 = \left(\sum_{i_{\text{even}}} a_i - \sum_{i_{\text{oneven}}} a_i\right) \bmod 11$$

a_i is i^{e} cijfer van a

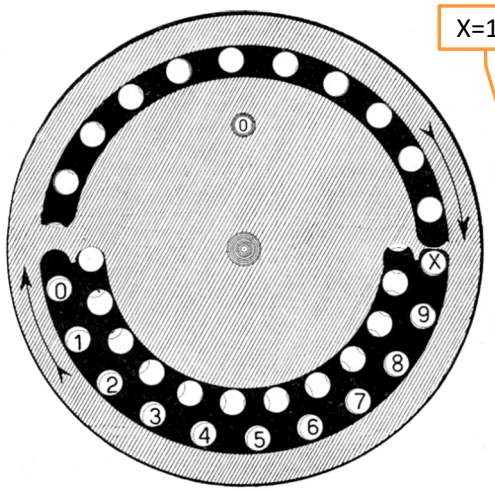
$\circledast$ is $+$ of $-$ of $\times$ of $\div$

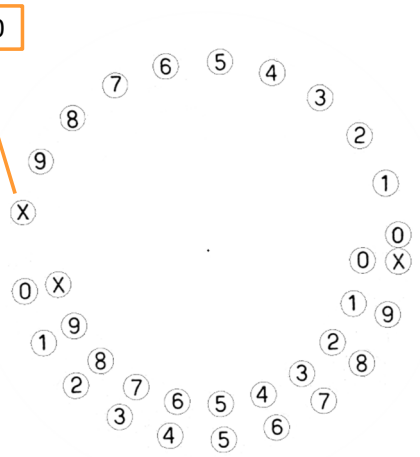
$$a \circledast b = c \qquad (a \bmod 11) \circledast (b \bmod 11) = c \bmod 11$$

Calculator:

Controlex

ReumHelm BV, 1953





Creditcard

Doel: controle van getal a

$$\left(\sum_{i=0}^{11} \begin{cases} i \text{ odd:} & 2a_i \bmod 9 \\ i \text{ even:} & a_i \end{cases}\right) \bmod 10 = 0$$

a_i is i^e cijfer van a

De wiskunde:

gewicht: w_i — i^e cijfer van a: a_i

$$\left(\sum_i w_i a_i\right) \bmod p = 0$$

- 1 van p willekeurige fouten wordt gemist
- maar fouten zijn niet willekeurig!

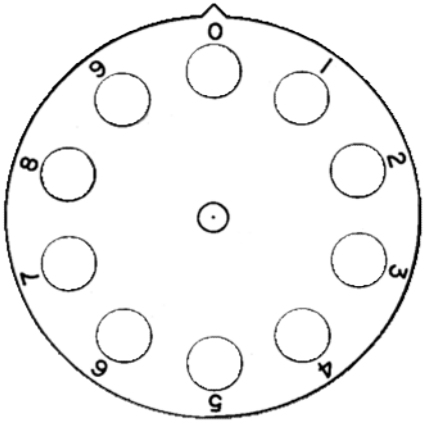
Calculator:

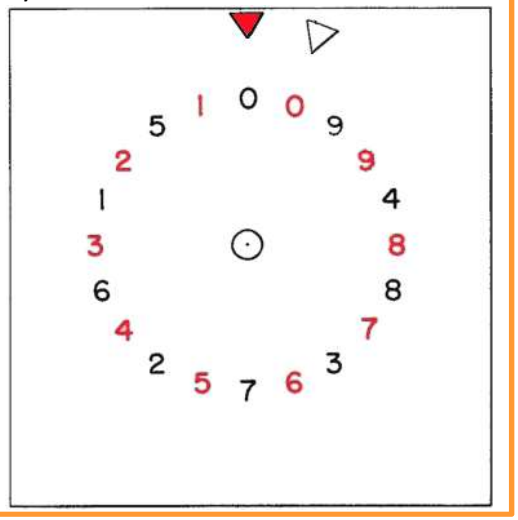
L.C. Zitnik (Pure Oil Comp.) 1959

Algoritme:

Hans P. Luhn (IBM) 1954

Hans P. Luhn





Verbeterde formule

Doel: controle van getal a

$$(3a_1 + 6a_2 + 8a_3 + 5a_4 + 7a_5 + 10a_6 + 2a_7 + 4a_8 + 9a_9 + a_{10}) \bmod 11 = 0$$

a_i is i^e cijfer van a

a_{10} is check digit

De wiskunde:

gewicht | i^e cijfer van a

$$\left(\sum_i w_i a_i\right) \bmod p = 0$$

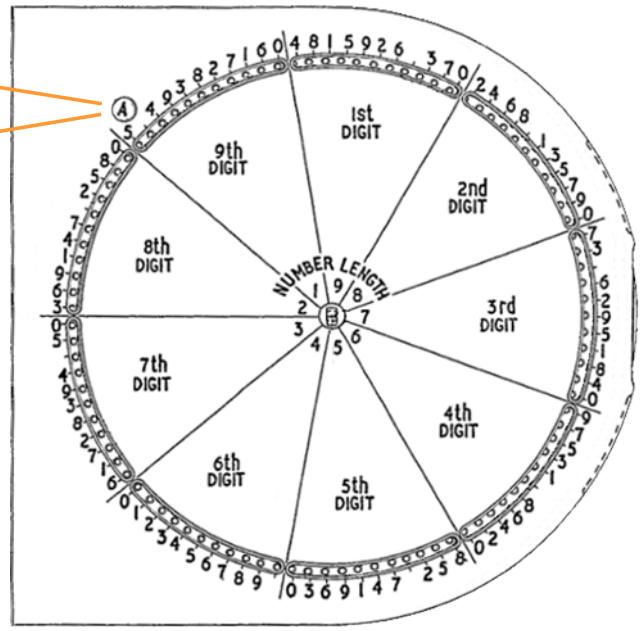
- verschillende gewichten voor elke positie i
- priemgetal p is beter!

Calculator:

A. H. F. Richardson, 1965

a_{10}

A=10



Verhoeff-algoritme

Doel: controle van getal a

$\sigma(x)$ is permutatie

a_i is i^e cijfer van a

$$\sigma(a_n) \otimes \sigma^2(a_{n-1}) \otimes \ldots \sigma^n(a_1) \otimes a_0 = 0$$

$\otimes$: vermenigvuldiging in puntgroep D_5

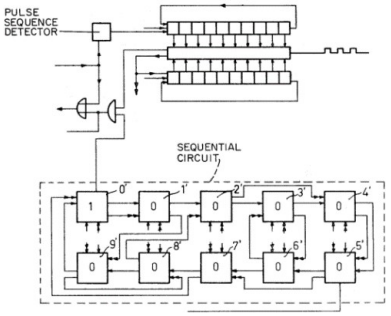
De wiskunde:

D_5	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

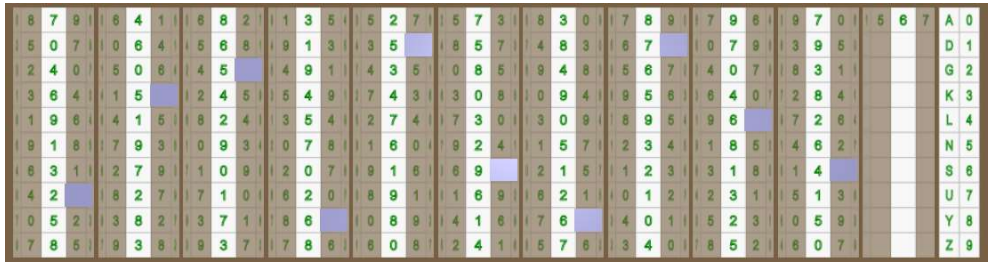
x	0	1	2	3	4	5	6	7	8	9
σ	1	5	7	6	2	8	3	0	9	4
σ^2	5	8	0	3	7	9	6	1	4	2
σ^3	8	9	1	6	0	4	3	5	2	7
σ^4	9	4	5	3	1	2	6	8	7	0
σ^5	4	2	8	6	5	7	3	9	0	1
σ^6	2	7	9	3	8	0	6	4	1	5
σ^7	7	0	4	6	9	1	3	2	5	8
σ^8	0	1	2	3	4	5	6	7	8	9

Calculator:

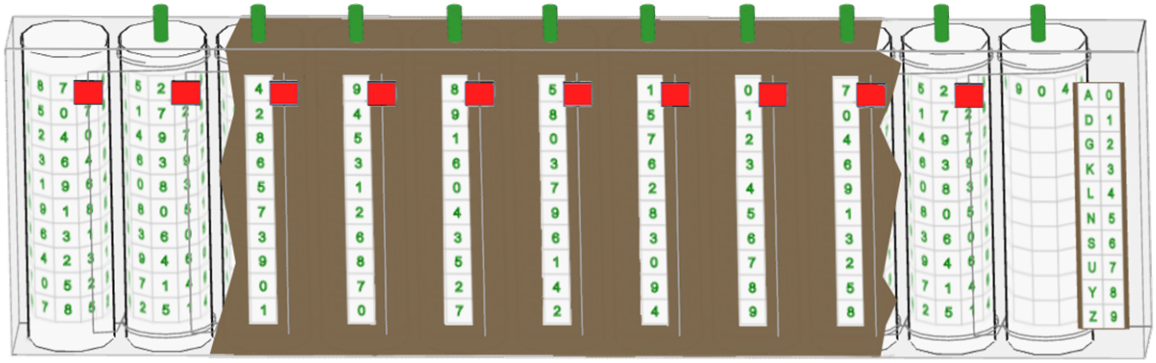
J. Verhoeff (Philips), 1969



Andries de Man, 2017



Verhoeff



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9

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9	8	7	6	5	1	2	3	4	0

2

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1

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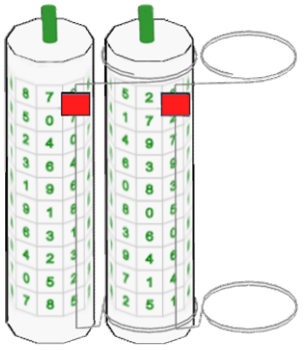
0 = rechts

0	4	3	2	1	5	6	7	8	9

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K	3
L	4
N	5
S	6
U	7
Y	8
Z	9