Adriaen Vlacq and Ezechiel de Decker: Dutch Contributors to the Early Tables of Briggsian Logarithms

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Introduction

The history of logarithms has by now progressed to a state where the logarithm is just one of the few hundred items in a standard book of mathematical functions.

How different it used to be during the past few centuries, when the logarithmic table was a powerful "calculating instrument"! And how fast this shining computational miracle has been eclipsed, together with the slide rule, by the electronic calculator of the 1970s. When Henry Briggs (1560-1630) took over from John Napier (1550-1617) the concept and further tabulations of the logarithm in 1615, he was to start a computing revolution in multiplication, division, and powers by using of logarithmic tables. However, in the 1620s three other men had made their own contributions to the early Briggsian tables:

- Edmund Gunter (1581-1626), Professor of Astronomy beginning in 1619 at Gresham College, London, and very involved in navigation at sea. He was a good friend of Briggs, who had been Professor of Geometry at the same college.

- Ezechiel de Decker (1603-1647), who was from 1621 to 1629 a teacher of geometry and arithmetic, especially for commerce and accounting, and surveyor in Gouda. This city was one of the seven provinces in the *Republiek van de Vereenighde Nederlanden* (Republic of the United Netherlands).

- Adriaen Vlacq (1600-1667), born in Gouda from a family of merchants and magistrates, well educated at the Latin School, but without a University degree. In his early twenties Vlacq became entranced by mathematics, and he probably met de Decker while educating himself in the art of arithmetic and geometry.

The major achievements of these four players in the game of calculating and publishing tables of 10-based (Briggsian) logarithms are summarized in Table 1.[1]

Ref	Year	Author	Title	NUM Precision	TRIG Precision
[1]	1617	Henry Briggs (England)	*Logarithmorum Chilias Prima*	Numbers 1 to 1,000: † 3 dec. in x 14 dec. out	—
[2]	1620	Edmund Gunter (England)	*Canon Triangulorum*	1 to 1,000: 3 x 7 dec.	1 min. x 7 dec. †
[3]	1624	Henry Briggs (England)	*Arithmetica Logarithmica*	1 to 20,000 and † 90,000 to 100,000: 4(+) x 14	—
[4]	1627	Ezechiel de Decker (Holland)	*Nieuwe Tel-Konst Part I, plus attached booklet with tables* derived from Ref. 2 & 3	1 to 10,000: 4 x 10 dec.	1 min. x 7 dec.
[5]	1627	Ezechiel de Decker (Holland)	*Nieuwe Tel-Konst Part II*	1 to 100,000: † 5 x 10 dec. (the "Great Table")	—
[6]	1628	Adriaen Vlacq (Holland)	*Arithmetica Logarithmica Part II*	1 to 100,000: † 5 x 10 dec. (the "Great Table")	1 min. x 10 dec. †
[7]	1633	Henry Briggs (England)	*Trigonometria Britannica*	—	1/100 min. † x 14/10 dec.
[8]	1633	Adriaen Vlacq (Holland)	*Trigonometria Artificialis*	1 to 20,000: 4(+) x 10 dec.	10 sec. x 10 dec. †

Table 1 : Main publications of early tables of Briggsian logarithms.
†indicates contribution by actual computations.

[1]Note: if we consider a logarithmic table as a transformation from a number to its logarithm, then we can describe the input precision of a table of numbers 1 to 1,000 as "3 decimals". Hence the precision notation"3 × 14" in the first entry of table 1, which means 3 decimals in, and 14 decimals out (in the "mantissa", the part after the decimal point). This is a better notation because the table is not literally restricted to the numbers 1 to 1,000: any position of the decimal point is allowed, as indicated by the "characteristic" of the logarithm (the part before the decimal point).

Cooperation Between Vlacq and de Decker:
The Facts

We will tell how Adriaen Vlacq and Ezechiel de Decker extended Briggs' *Arithmetica Logarithmica* [3] into a complete table of the numbers 1 to 100000.

Much has been written about the strained relationship between these two Dutchmen, because their early cooperation surely was not executed as intended.

The following documented events tell the story:

1. **1625:** Vlacq happens to read Briggs' *Aritmetica Logarithmica*, and decides to publish a completed version ("Great Table") of this book, see preface of [6].

2. **1625, Dec. 24:** Vlacq obtains copyright for his planned publication, see [21].

3. **1625, Dec. 27:** Vlacq signs contract with de Decker to have him calculate the still missing parts for the "Great Table", see [21].

4. **1625, Dec. 29:** Vlacq contracts Rammaseyn to print the "Great Table", see [21].

5. **1626, Sept. 4:** de Decker publishes the *Eerste deel van de Nieuwe Telkonst* about non-logarithmic arithmetic, [4a], with reprints of existing logarithmic tables, [4b].

6. **1626, Dec. 31:** Vlacq summons de Decker by notary to finish calculations, see [21].

7. **1627, Oct. 2:** de Decker has prepared for publication the *Tweede deel van de Nieuwe Telkonst*, containing the "Great Table", see [5] and [22].

8. **1628:** Vlacq publishes *Arithmetica Logarithmica, Ed. 2*, containing the "Great Table", including new trigonometrical logarithms, see [6].

The surprising aspect of fact 7 is that the one known copy of *Tweede deel van de Nieuwe Telkonst* was discovered only in 1920, see [22]. This means that before 1920 all research on early tables was based on incomplete information, resulting today in a state of confusion: some people claim that de Decker was literally the first to print a complete logarithmic "Great Table" of 1 to 100,000, because of fact 7. Others, having used research results from before 1920, still believe Vlacq was the first. And then we have the majority who give credit for the "Great Table" to both Dutchmen equally, given the compelling argument:

Without either of the two, the "Great Table" would not have materialized in Gouda. Vlacq provided the first initiative, his business sense, financial funding, translation of Briggs' texts from Latin, some of the calculations, the new trigonometrical tables and the successful academic 1628 edition [6] of the "Great Table" as part II of Briggs' work, while de Decker did the major part of the calculations and finalised the "Great Table" proper, although his commerce-oriented 1627 edition [5] did not reach the public.

What may actually have happened: fact and fiction

When we try to imagine the full story of Vlacq and de Decker, we might agree on the following scenario to complement the facts above.

Fact 1

Privileged with knowledge of the Latin language, Vlacq gets acquainted with Briggs' *Arithmetica Logarithmica* at the age of 25; he is not so sure about his own mathematical abilities, and consults some mathematicians or arithmeticians, including de Decker, on the potential usefulness of a fully completed "Great Table". His business instinct tells him that publishing a table for the European continent, in a living language like French or Dutch, with a full range of numbers of 1 to 100,000—without a gap—and swiftly executed by suppressing some precision from Briggs' 14 decimals in the calculations, could be a worthwile and profitable investment. His conscience does not trouble him with possible plagiarism because, in his introduction to [3], Briggs has actually invited others to execute the calculations to resolve the gap in his *Arithmetica Logarithmica*. But Vlacq did not inform Briggs of his plan (which Briggs also had asked in his invitation); in the preface to his own publication [6], he describes in detail the above-mentioned motives for his approach, adding the altruistic remark this would ensure "sufficient copies to be available for this country, and also in another language than Latin".

Fact 2, 3, and 4

Vlacq's business sense makes him legally contract partners for the project, and obtain copyright ("Privilegie") from the authorities ("Staten Generaal") to publish the "Great Table" for ten years.

Fact 5

Part of the marketing plan was to include a preannouncement of the "Great Table" and some preliminary (already existing) logarithmic tables in de Decker's *Nieuwe Tel-Konst* Part I. This book contained Napier's texts on *Rabdologia* (Napier's bones), as translated by Vlacq into Dutch, and de Decker's own text on commercial arithmetic; a copy of Simon Stevin's *De Thiende* on decimal numbers was attached. This "non-logarithmic" book was combined with a separate booklet, *Nieuwe Telkonst, inhoudende de logarithmi* [4b], containing the logarithms of 1 to 10,000 copied from Briggs [3], and the trigonometric logarithms copied from Gunter [2]. In each of these books, the introduction announces the future publication by Vlacq of the "Great Table" (*Het Groote Werck*) in three languages: Latin, French, and Dutch.

Fact 6

One of the safeguards of the project was the extensive guide for computing the missing numbers, which Briggs gave in chapter XII and XIII of his *Arithmetica Logarithmica*. However, the calculations following Briggs' guidelines proved to be too time-consuming. Vlacq knew that

Briggs was working on his own version of a "Great Table", and in panic he summons de Decker by notary to finish the agreed calculations before May 1, 1627, see [21]. He even offers his own assistance to provide half of the remaining calculating effort, if necessary. Now we follow Bruins' hypothesis [25], that de Decker discovers a shortcut to the lengthy calculations. Briggs' directions required one first to determine all primes in the gap from 20,000 to 90,000, calculate their logarithms, and from there work to the remaining numbers. De Decker decides to use a different strategy of extrapolation and interpolation: he first determines—in the missing region—the logarithms of all multiples of Briggs' numbers by adding log(2), log(3), etcetera, to their known logarithms, and then he determines the remaining logarithms by interpolation, greatly assisted by the decrease in precision from 14 down to 10 decimals. This hypothesis seems to be supported by Vlacq's introduction to his *Arithmetica Logarithmica*, Part II, refering to Briggs' high estimate of the work effort needed and claiming the credit for himself: "...so I adjusted the method to find easily every logarithm with not more than 10 decimals...".

TWEEDE DEEL
VANDE
NIEVWE TEL-KONST
OFTE
VVONDERLIICKE
KONSTIGHE TAFEL,
Inhoudende de Logarithmi, voor de Getallen van 1 af tot 100000 toe.
Eerst ghevonden van IOHANNE NEPERO Heer van MARCHISTOVN, Shotsman.
WAER DOOR
DE MVLTIPLICATIE GHEDAEN
wort door Additie: de Divisie door Substractie:
de Reghel van Drien door Additie ende Substractie,
soo wel in ghebroken, als
heele Ghetallen:
Mitsgaders
Alle sware Arithmetische Questien, als, Rekeninghen van
Silver en Gout van Rabbatteren van Simpele ende Gecomposeerde Interesten:
d'Extractien; van Radix Quadraet, van Radix Cubicq, &c. met sonderlinghe lichticheyt tot verwonderingh ghesolveert
worden, diergelijcke voor desen noyt
ghehoort ofte ghesien.
Seer dienstich, profijtelijck ende vermakelijck, voor de Cooplieden, ende Liefhebbers van de
Arithmetische ende Mathematische Konsten.
DOOR
EZECHIEL de DECKER, Reken-Meester, Landt-Meter,
ende Lief-hebber der Mathematische Konst, residerende ter Goude.
TER GOVDE,
By Pieter Rammaseyn, Boeck-verkooper
Inde Korte Groenendael, in't vergult A,B,C.
M. DC. XXVII.
Met Privilegie voor thien Iaren.

Figure 1: Title Page of de Decker's 1627 version of the "Great Table".

Fact 7

De Decker is so pleased with finishing the calculations by his own method that he feels himself freed of his contract with Vlacq, and immediately asks Rammaseyn to prepare the printing plates for the "Great Table". He had already written the text for part II of his *Nieuwe Tel-Konst* in Dutch, which consisted of a very practical guide to use the logarithmic tables for commercial purposes (including compound interest calculations). His preface is written in "we" form, implying Vlacq's assent, and the distinction between this publication and Vlacq's later one, is logically explained, see [5]. Again the future publication by Vlacq is announced in the introduction.

ARITHMETIQVE
LOGARITMETIQVE
OV
LA CONSTRVCTION ET
VSAGE d'VNE TABLE CONTENANT
les Logarithmes de tous les Nombres depuis l'Vnité jusques à 100000.
ET
D'VNE AVTRE TABLE EN
laquelle sont comprins les Logarithmes des Sinus,
PAR LE MOYEN DESQVELLES ON RESOVLT TRES-FACILEMENT les Problemes Arithmetiques & Geometriques.
CES NOMBRES PREMIEREMENT
sont inventez par IEAN NEPER Baron de
LA DESCRIPTION EST TRADVITE DV LATIN EN
DIEV NOVS A DONNÉ L'VSAGE DE LA VIE ET D'ENTENDEMENT, PLVS QVIL N'A FAIT
PAR LE TEMPS PASSÉ.
A GOVDE,
Chez Pierre Rammasein.
M. DC. XXVIII.

Figure 2: Title Page of Vlacq's 1628 publication (in French) of the "Great Table".

Did Vlacq know? If not, he was bound to hear soon. Vlacq, de Decker, and their printer, Rammaseyn, were all living within walking distance of each other in Gouda, a mid-sized town in Holland, where secrets would not have been kept long in its community. We know that de Decker's book never reached the public, as only one copy has ever been discovered (in 1920). Therefore it seems likely that Vlacq and de Decker must have "agreed" to suppress publication of the *Nieuwe Tel-Konst*, Part II, not only because it violated their contract, but also because Vlacq must have shown that an "academic" version (in the style of Briggs' *Arithmetica Logarithmica*) would sell better than de Decker's competing "merchant-oriented" version. This proved right after all, because logarithmic tables have never really taken off in the commercial world during the succeeding 300 years. A few copies (at least the one found in 1920) of de Decker's *Nieuwe Tel-Konst*, Part II, may have been printed in full, probably for sen-

timental reasons of de Decker, but the complete set of the tables printed by de Decker were bound into Vlacq's 1628 issue of *Arithmetica Logarithmica*, Part II (the print image of Vlacq's "Great Table" is identical to the one in de Decker's version).

Fact 8

The full title of Vlacq's 1628 publication was *Arithmetica Logarithmica, Editio 2, aucta per A. Vlacq*, where "aucta" stood for the extension of the tables between 20,000 and 90,000. He included an abridged version of the extensive introduction by Briggs, carefully omitting his directions for calculation of logarithmic tables by the method of differences in the original Chapters XII and XIII. He also added a very useful table of trigonometric logarithms.

Vlacq gives in his introduction due credit to Napier and Briggs, but he must have been so annoyed by de Decker's attempted breach of contract, that he does not mention the contribution by de Decker at all (while de Decker has always gracefully acknowledged Vlacq in his own books). Actually it suited Vlacq well to pose as the only author, next to Napier and Briggs. The original intention was to publish "the Great Work" in three languages, but only Latin and French copies have survived. There are indications [11] that Dutch copies were indeed printed, but moved to England later, where the table pages were used in other books, like an English version of [6], published by George Miller in London, 1631.

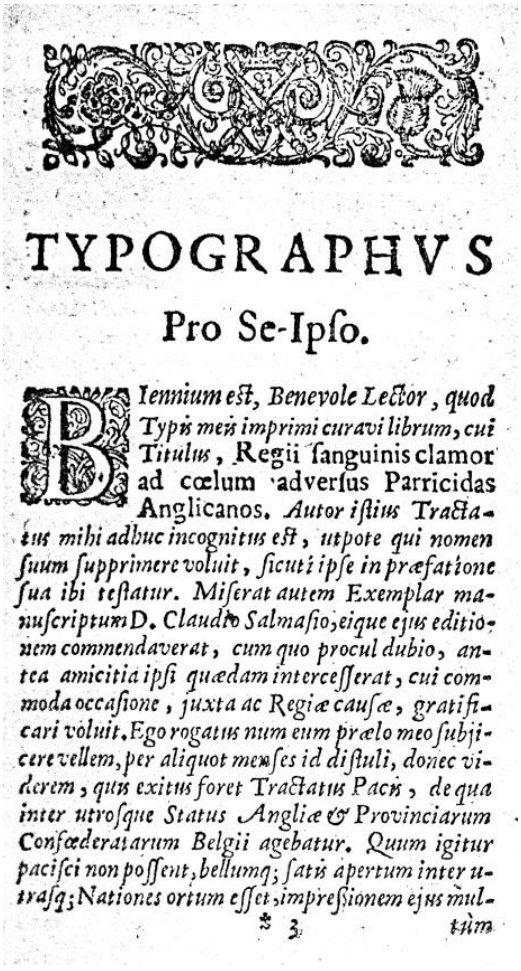
Trigonometric Tables

The massive number of calculations needed in astronomy was the primary motive for inventing the logarithm. That is the reason why Napier started with logarithms of sines. But many equations in trigonometry require the logarithms of both numbers and sines, tangents, etc. Briggs started with calculating logarithms of numbers for his *Arithmetica Logarithmica*, and finally in 1633 his work on trigonometric logarithms was published [7]. It was actually Edmund Gunter who published the first combined logarithmic table of sines, tangents, and numbers in his *Canon Triangulorum* (1620, see [2]). He copied the logarithms of numbers from the Chilias Prima of his friend and colleague Briggs, but the logarithms of the sines and tangents must have been calculated by himself, possibly with some help from Briggs. It should be noted—to his credit—that Vlacq in his 1628 *Arithmetica Logarithmic*, Part II, had added trigonometric tables, and with a higher precision than Gunter had published in 1620 (10 decimals instead of Gunter's 7 decimals). This means that Vlacq must have recalculated (by himself, or with others) the logarithms of sines and tangents, because his own logarithms of numbers had not enough precision to be used for table look-up. This was a computational accomplishment that has been largely neglected by most researchers. Another large effort must have been spent by Vlacq to construct the trigonometric tables (per 10 seconds of arc) in his 1633 *Trigonometria Artificialis*. This work was very close, and even competitive, to Briggs' last table, the *Trigonometria Britannica* of that same year, which gave trigonometric logarithms to a higher precision of 14 decimals for sines (10 decimals for tangents), but by "centesimal" intervals of 1/100 degree. Briggs (and also Gunter, although his *Canon Triangulorum* had been sexagesimal) were early advocates of the centesimal system for angle units, but the world was not yet ready for that. It appears that Vlacq's sexagesimal-based table was more popular with the public. Some researchers have turned this issue upside-down, claiming that Vlacq, by publishing his sexagesimal tables, blocked acceptance of the centesimal units at that time (sic!). Suspicions that Vlacq copied in his own book parts from Briggs' tables, are unfounded as the sexagesimal and the centesimal ranges do not coincide except in some whole minutes. It should be noted that Gellibrand's extensive introductory chapters in *Trigonometria Britannica* have been reused in Vlacq's *Trigonometria Artificialis* (with or without Gellibrand's agreement).

Vlacq After 1628

Vlacq lived in Gouda until 1633, working probably on the construction of the new tables in the *Trigonometria Artificialis* of 1633. In 1632 he published the *Ephemerides Motuum Coelestium* [6d], with tables for the positions of stars and planets for the years 1633 to 1636. The tables were taken from *Tabulae Motuum Coelestium* by the Flemish astronomer Philip van Lansberg and from Kepler's *Rudolphine Tables*, but Vlacq edited the introduction and the instructions for use. After 1633 Vlacq worked only as book-publisher and seller, and until his death in 1667 he never applied himself again to the art of mathematics (or astronomy), proving again that he was not really a scientist. We know that Briggs learned (within a few months) of the publication of Vlacq's *Arithmetica Logarithmica*, Part II, because his reaction, disappointed but not angry, has survived in a letter to John Pell, see [6a]. Others were more explicit in blaming Vlacq's "ill dealings", like Richard Norwood in the preface to his 1631 *Trigonometrie*, see [9]. It is not known whether Vlacq ever had direct contact with Briggs, but he certainly communicated with Henry Gellibrand (Gunter's successor at Gresham College after 1627) who assumed the responsibility of publishing Briggs' *Trigonometria Britannica* after he died in 1630. This resulted in Vlacq's traveling to London, negotiating with Gellibrand, and arranging for Rammaseyn to print Briggs' last work in Gouda, together with his own *Trigonometria Artificialis*. He stayed in London as bookseller until the conflict between King Charles I and Parliament turned into civil war and forced him to leave England in 1642. Being a dexterous businessman, and a foreigner too, he was not always popular among his bookseller colleagues, one of whom accused him of: "Lurking here, observing what is most useful and vendible, and causes it forthwith to be printed abroad", see [20]. He then went to Paris, again as bookseller, in the rue St. Jacques. After another conflict in Paris, he returned to Holland in 1648. Around 1650 Vlacq estab-

lished himself in The Hague as book publisher and seller under the name "Sumptibus Adr. Vlac", later "Ex Typografia Adr. Vlac". He also acquired his own print shop, which he bought in 1653 from Johannes Rammazeyn (the son of Vlacq's old printer Pieter Rammaseyn in Gouda!). This was his most productive period during which he published and printed works of many authors, including some famous scholars like Hugo Grotius, Christiaan Huygens, Gerardus Johannes and Isaak Vossius. In 1653 and after, he even published The Hague's first newspaper, *Het Wekelijcke Nieus*, the descendant of which became *De Haagsche Post* (today *HP/De Tijd*). One of his conflicts in 1654, with John Milton (*Paradise Lost*) while publishing some of his polemic pamphlets, stimulated him to write a defense for himself against Milton's and other accusations he had suffered during his professional life, titled *typographus pro se ipso* (the bookseller's defense). In this short paper of 12 pages, Vlacq cites Milton's defamations and stands up to the accusations, adding a description of his bookseller's life from his own perspective, see [24]. Again he carefully avoids in this text any mention of de Decker, who had died in 1647.



TYPOGRAPHVS

Pro Se-Ipſo.

Biennium eſt, Benevole Lector, quod Typis meis imprimi curavi librum, cui Titulus, Regii ſanguinis clamor ad cœlum adverſus Parricidas Anglicanos. Autor iſtius Tractatus mihi adhuc incognitus eſt, utpote qui nomen ſuum ſupprimere voluit, ſicuti ipſe in præfatione ſua ibi teſtatur. Miſerat autem Exemplar manuſcriptum D. Claudio Salmaſio, eique ejus editionem commendaverat, cum quo procul dubio, antea amicitia ipſi quædam interceſſerat, cui commoda occaſione, juxta ac Regiæ cauſæ, gratificari voluit. Ego rogatus num eum prælo meo ſubjicere vellem, per aliquot menſes id diſtuli, donec viderem, quis exitus foret Tractatus Pacis, de qua inter utroſque Status Angliæ & Provinciarum Confœderatarum Belgii agebatur. Quum igitur paciſci non poſſent, bellumq; ſatis apertum inter utraſq; Nationes ortum eſſet, impreſſionem ejus multùm

* 3

Figure 3: First Page of Vlacq's defence against Milton, et al.

A most interesting part in *typographus pro se ipso* might raise some second thoughts on his presumed business talent at publishing logarithmic tables:

> "What costly expenditures, what labor, how much time I spent on these three books, besides others which I completed at that time, they can judge who know them. This I can testify: I drew from them much more financial loss than gain. Yet that does not trouble me, for I know that I have done a work most welcome to many learned mathematicians, and that those books will in a few years be held in great esteem, and that I shall receive some thanks from posterity, just as some erudite men have already made honorable mention of me in their books."

This reveals a little more about Vlacq's vanity, but the prediction about his tables proved to be right!

In the Hague, Vlacq published between 1651 and 1667 a number of reprints of the tables from *Arithmetica Logarithmica*, Part II, but in a completely different form: the tables had been reduced to a precision of seven decimals, to the number range 1 to 10,000, and to trigonometric tables per minute of arc. The book containing these "small" tables, was now octavo-bound in a very portable format (some 11 by 17 cm, or even slightly smaller). This was the start of the long and successful series of the "Vlacq table" publications over the next 150 years. It is remarkable that between 1651 and 1667 some of the "small" Vlacq tables were printed by other printers, although Vlacq did have his own printing shop during this period (see table 2 of those copies that still exist in libraries).

Year	Language	Printer	City
1651	French	chez Adrian Vlacq	The Hague
1651	Latin	apud PhilippVm de Croy	Leiden
1657	Dutch	by Pieter Goos	Amsterdam
1658	Dutch	by Pieter Goos	Amsterdam
1661	Dutch & Latin	by Adriaen Vlacq	The Hague
1665	Dutch	by Hendrick Doncker	Amsterdam
1665	Latin	Ex Typographia Adriani Vlacq	The Hague
1666	French	chez Adrian Vlacq	The Hague

Table 2: "Small" Vlacq tables published between 1651 and 1667.

De Decker after 1628

After his involvements with logarithmic tables de Decker moved to Rotterdam in 1629, where he became involved in teaching sea navigation and in mediating maritime conflicts [21]. In 1631 he published a commerce-oriented book on compound interest calculations [6b], and in 1632 a handbook for navigation at sea [6c]. Both these books he had printed by his old printer, Pieter Rammaseyn in Gouda. His second book, *Practyck van de groote zee-vaert*, lacked any reference to the "Great Table" in either his own *Nieuwe Telkonst*, Part II, or in Vlacq's *Arithmetica Logarithmica*, Part II; de Decker probably avoided the sensitive issue by using the older

"small" tables from his *Nieuwe Telkons*, Part I. Around 1640 de Decker moved to The Hague, where he took on the profession of wine gauger until his death in 1647.

Posthumous Editions of Vlacq's "Small" Tables

After Vlacq died in 1667, many reprints were published, under his imprint, of the small pocket-sized tables (numbers from 1 to 10,000). This indicates the stature of his name on the Continent. The reprints have been published in four different languages, not very important for the numerical tables but relevant for the title pages and the 30 to 40 pages of introductory text, consisting of instructions for use, four additional chapters on plane and spherical trigonometry (largely derived from Gellibrand) and an annex on astronomical questions (sometimes even on interest calculations).

The Latin version, published first in Lyon (Jean Thioly), Amsterdam (family Boom) and after 1700 mostly in Frankfurt (Johann Friedrich Fleischer), was often called:

Tabulae sinuum, tangentium, et secantium, et logarithmi sinuum, tangentium, et numerorum ab unitate ad 10000

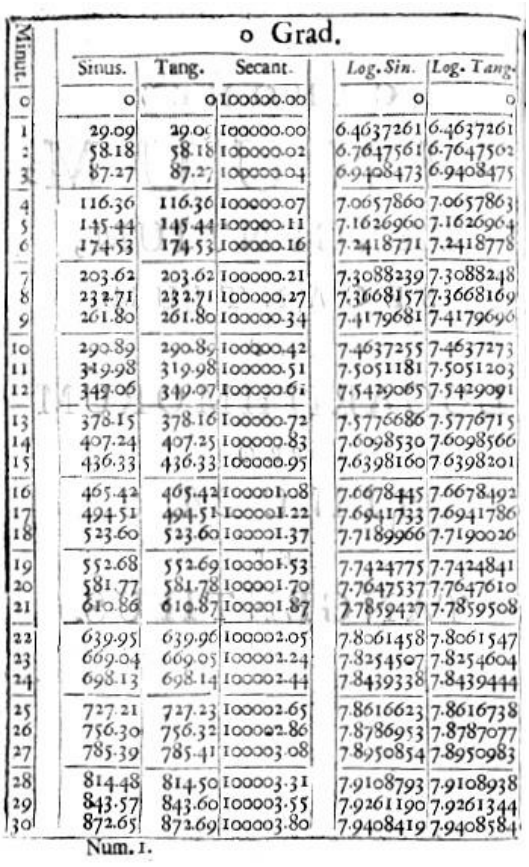
TABULÆ
SINUUM,
TANGENTIUM,
ET SECANTIUM,
ET
LOGARITHMI
SINUUM, TANGENTIUM,
& Numerorum ab unitate ad 10000.
Ab A. VLACQ.
Editio ultima ab innumeris mendis, quibus postrema scatebat, purgata.
AMSTELÆDAMI,
Apud JANSSONIO-WAESBERGIOS.
cIↃ IↃCC XLII.

Figure 4: Title Page of one of the many "small" tables by Vlacq (1742).

It is not clear if after Vlacq's death a Dutch book, containing only the small tables, was ever been published under the name Vlacq, but his tables certainly have been used as annexes in numerous Dutch handbooks on navigation at sea (see also the following section), for example under the name:

De Taeffelen der sinuum, tangentium en secantium, ofte der hoeckmaeten, raecklijnen en snylijnen, alsmede de logarithmi der hoeckmaeten, raecklijnen en snylijnen, en achter de selve de logarithmus numeri van 1 tot 10,000

(Note that some of the Dutch publications had the logarithms of secants ("snylijnen") added, in addition to Vlacq's standard set of logarithms for sines and tangents).


o Grad.

Minut.	Sinus.	Tang.	Secant.	Log. Sin.	Log. Tang.
0	0	0	100000.00	0	0
1	29.09	29.09	100000.00	6.4637261	6.4637261
2	58.18	58.18	100000.02	6.7647561	6.7647502
3	87.27	87.27	100000.04	6.9408473	6.9408475
4	116.36	116.36	100000.07	7.0657860	7.0657863
5	145.44	145.44	100000.11	7.1626960	7.1626964
6	174.53	174.53	100000.16	7.2418771	7.2418778
7	203.62	203.62	100000.21	7.3088239	7.3088248
8	232.71	232.71	100000.27	7.3668157	7.3668169
9	261.80	261.80	100000.34	7.4179681	7.4179696
10	290.89	290.89	100000.42	7.4637255	7.4637273
11	319.98	319.98	100000.51	7.5051181	7.5051203
12	349.06	349.07	100000.61	7.5429065	7.5429091
13	378.15	378.16	100000.72	7.5776686	7.5776715
14	407.24	407.25	100000.83	7.6098530	7.6098566
15	436.33	436.33	100000.95	7.6398160	7.6398201
16	465.42	465.42	100001.08	7.6678445	7.6678492
17	494.51	494.51	100001.22	7.6941733	7.6941786
18	523.60	523.60	100001.37	7.7189966	7.7190026
19	552.68	552.69	100001.53	7.7424775	7.7424841
20	581.77	581.78	100001.70	7.7647537	7.7647610
21	610.86	610.87	100001.87	7.7859427	7.7859508
22	639.95	639.96	100002.05	7.8061458	7.8061547
23	669.04	669.05	100002.24	7.8254507	7.8254604
24	698.13	698.14	100002.44	7.8439338	7.8439444
25	727.21	727.23	100002.65	7.8616623	7.8616738
26	756.30	756.32	100002.86	7.8786953	7.8787077
27	785.39	785.41	100003.08	7.8950854	7.8950983
28	814.48	814.50	100003.31	7.9108793	7.9108938
29	843.57	843.60	100003.55	7.9261190	7.9261344
30	872.65	872.69	100003.80	7.9408419	7.9408584

Num. 1.

Figure 5: Page from Vlacq's "small" trigonometrical table (1742).

The French version was published in Amsterdam by Doncker, and until 1699 in France (Jean Thioly in Lyon and Jean Jombert in Paris). It had the title:

Tables de sinus, tangentes et secantes et de logarithmes des sinus, tangentes, et des nombres depuis l'unit jusques 10000

The German version was first published in Amsterdam by Johannes van Ravensteijn, and also by the family Boom, but after 1700 it was reprinted many times in Frankfurt and Leipzig by J.F. Fleischer. Its title was:

Tabellen der Sinuum, Tangentium und Secantium: wie auch der Logarithmorum vor die Sinus, Tangentes, und die Zahlen von 1 bis 10000

This version was improved in the 1748 version by Johann Jacob Hentschen, and in the 1789 version by Joh. Jac. Ebert. The very last version, published by Fleischer in 1821, was restructured by Gottlob Nordmann. Those tables looked very different from the ones that had been

stable since the 1650s, now containing explicit difference columns and additional square, cube, and root tables. The original introduction, unchanged for more than 150 years, was completely replaced by a new introduction and a user manual.

LATIN	FRENCH	GERMAN
	1668	
1670	1670	
		1673
1681		
	1683	
		1689
	1690	
		1695
	1699	
1706		1706
1721		1721
		1725
1726		1726
		1732
1738		1738
1742		
1748		1748
1757		1757
		1763
1767		1767
1768		1768
1775		1775
1778		1778
1784		
		1789
1790		1790
1808		1808
		1821

Table 3: Editions of tables "by Vlacq" after Vlacq's death in 1667.

Table 3 shows that a very large number of posthumous "Vlacq" tables were published after 1700 in German or Latin, almost all of them by Fleischer in Frankfurt (only the Latin versions of 1721, 1742, and 1784 were printed in Amsterdam, by J. Boom, Janssonius-Waesbergius, and Schouten, respectively).

There must have been tens of thousands of "small" Vlacq tables in the 18th century, especially in Germany. This explains why Vega added in 1794 the reference "nach Vlack" ("following Vlack") to the title page of his famous table *Thesaurus Logarithmorum Completus*.

Some other Contributors to Early Logarithmic Tables

Vlacq and de Decker were not the only Dutchmen who were actively involved in logarithmic tables. Christiaen Huygens, one of the inventors of the pendulum clock, published in 1661 a method for fast calculation of logarithms by approximation, based on "squaring an equilateral hyperbola", see [25]. It is not known if this method ever was used. Some Dutchmen in the 17th century reproduced "small" Vlacq tables in new books, like Dirk Rembrants van Nierop (1671), Claas Jansz Vooght (1685), Johannes van Keulen (1698), and Christiaan Wolff & Nicolaas Epkema (many editions between 1711 and 1780). Also the well-known handbooks for navigation at sea by Abraham de Graef (*De Seven Boecken van de Grote ZeeVaert*, 1657) and Klaes Hendriksz Gietermaker (*'t Vergulde Licht der Zee-vaert, ofte Konst der Stuurluyden*, 1671) contained Vlacq's tables of logarithms among the many other navigational and astronomical tables, see [14].

During the 17th and 18th century more than a hundred logarithmic tables were published, see [19], which were derived–completely or partially—from the Briggs/de Decker/Vlacq computations. Many of those even had the print image of the "small" Vlacq tables, but later editions were provided with improved and more "user-friendly" table layouts. No extensive new calculations were done after the 1620s, until Georg von Vega in Austria and Gaspard de Prony in France recalculated logarithms in the 1780s.

Other scientists on the Continent were closely involved with trigonometrics and table construction, but did not contribute directly to Briggsian tables. For example, Willebrord Snellius calculated a new non-logarithmic trigonometric table (another *Canon Triangulorum*) around 1626, just before he died, but apparently he was not acquainted with logarithms. Kepler on the other hand, was keenly aware of Napier's publications, and he published limited logarithmic tables, e.g., in his *Rudolphine Tables*(1627), but these were still Napierian, not Briggsian.

Precision of logarithmic tables

Under Table 1 the precision of input and output of logarithmic tables, in number of decimals, was already mentioned. Generally speaking, mathematicians had a tendency to increase the output precision from Briggs' original 14 decimals, for example, to 28 decimals (of numbers 1 to 10,000) by E. Sang near the end of the 19th century, or even higher. But for practical use of a logarithmic table as a calculating instrument, a lower precision was allowable, and even advisable to keep the books within manageable size. Vlacq had started that trend by decreasing output precision first from Briggs' 14 to 10 decimals, and later in his "small" table to seven decimals, while reducing the input precision to four decimals (1 to 10,000). As a further example, Jeŕoṁe de La Lande produced in 1827 a table of four by five decimals precision, with even smaller size and weight than the 1972 electronic scientific calculator HP 35 (the "terminator" of logarithmic tables and slide rules alike). Such lower output precision provided a better balance with the input precision, because a relative overprecision in the logarithmic values would become lost in the transformation back to the number domain.

The "differences" between subsequent table entries (indicating the slope of the logarithmic function) are high in the lower regions and small in the higher regions, a characteristic of the "anti-exponential" curve. This

means that linear interpolation between two adjacent values, to get an additional decimal of input precision, gives less precise results in the lower regions. Therefore it is useful to extend tables with one additional input decimal in a lower region. For this reason many logarithmic tables have been published with input ranges extended to, for example, 108,000, or even to 200,000. At least one specimen of the *Arithmetica Logarithmica* exists with a range extension to 101,000, see [23].

Accuracy of Logarithmic Tables

Early handmade tables contained innumerable errors. The first Vlacq tables appear to have had hundreds of errors, but many were in the last decimal, which is not too bad for practical use, if the precision already covers ten decimals. Then the printer added his own "typos" in arbitrary decimals, which could be recognized as standing out in the gradual diminishment of the differences between successive entries.[2] It is not surprising that many reprints boasted the inclusion of new corrections, for example, Vlacq's 1742 edition, advertised as "Editio ultima ab innumeris mendis, quibus postrema scatebat, purgata" ("Ultimate edition, cleansed of the innumerable errors with which the last edition was infested"). The only useful aspect of errors in logarithmic tables has been that, as trace markers, they allowed literal copies of the tables to be detected in other publications, see [12]. We offer, as an example, the entry in seven-digit tables for the number 9482: it should have been rounded off to 9769000, but editors of different tables have copied many times the wrong entry 9768999.

Conclusion

Many disputes have been fought in the world of science over the question "who was first". For example, Napier was certainly the first to publish the concept of logarithms, but many researchers have tried to have him share more of the credit with Jost Bürgi, or even Longomontanus, Edward Wright, and the long list of those having discovered their own "addition of exponents". We have now seen the story of Briggs, Vlacq, and de Decker in more detail, and we conclude that the first logarithmic table of numbers from 1 to 100,000 was really completed by Vlacq and de Decker together, building on the initial computations of Briggs. In this endeavour, Vlacq's character is not without stains, as he never mentioned de Decker's contribution in any of his publications, not even his name! The first logarithmic tables for sexagesimal sines and tangents were computed by Gunter and, with higher precision, by Vlacq. For a very long time, all published logarithm tables have been based on the Briggs-de Decker-Vlacq and the Gunter-Vlacq computations. From a pure mathematician's perspective, Vlacq and de Decker did not contribute fundamentally to science with the "Great Table" (they have even been accused of "raiding Briggs' *Arithmetica Logarithmica*"). But we should realize that the fame of Vlacq was more due to the commercial publishing of the "Great Table", which was the source for most other tables until the late 1700s. Vlacq's other accomplishment is that, after the "Great Table", he perceived the public need for smaller and more practical tables that he started to produce, even though most "Vlacq" tables were produced after his death, and even though only 10,000 numbers were listed instead of the 100,000 in the "Great Table". The "small" Vlacq tables could just as well have been derived from the Briggs/Gunter tables as from Vlacq's own "Great Table".

Literature Sources

In the Netherlands, many Vlacq tables have been preserved in university and museum libraries, especially in Leiden and Amsterdam (about a hundred copies of some 40 different editions). Antiquarian bookshops often have Vlacq tables for sale, which also can shed more light on specific editions. At the end of the 19th century much research on the history of these logarithmic tables was carried out, mainly by Glaisher in England, see [10]-[12], and by Bierens de Haan in the Netherlands, see [13]-[16]. In the first half of the 20th century the discussion on this history has continued, certainly after the discovery in 1920 of the one and only copy of the *Nieuwe Telkonst*, Part II. Some erroneous or conflicting information exists, and has trickled down to derived publications or internet texts, like the overestimation of Vlacq's scientific capabilities, the role division between Vlacq and de Decker, or the misconception that Vlacq owned Rammaseyn's print shop in Gouda. Bierens de Haan has mentioned many published tables in [14] and in his—often disputed—list [16], some of which cannot be confirmed by library catalogs or other sources. Therefore these have been left out in this paper, like an isolated reference to a Dutch table in 1636. I have tried to use as much as possible the original references while compiling this history of the early Briggsian tables. But even a published statement by an author, like Vlacq, about himself, does not really prove its contents.

Acknowledgments

Many collector friends have given advice and remarks on this paper. The libraries consulted for source documents are those of the Universities of Utrecht, Leiden, and Amsterdam, the Museum Boerhaave in Leiden, the Koninklijke Bibliotheek in The Hague, and the Streekarchief Midden-Holland in Gouda.

References and Notes, in Chronological Order

[1] Briggs, H., *Logaritmorum Chilias Prima,* London, 1617.

[2] Gunter, E., *Canon Triangulorum, sive Tabulae Sinuum et Tangentium artificialium a Radium 10000.0000 & ad scrupula prime quadrantis,* London, W. Jones, 1620. Note: Gunter introduced in his table the addition of the term 10 to all trigonometrical logarithms (for example $sin(30°) = \text{-}0.3010300 + 10 = 9.6989700$), a convention that has persisted until today, although Gunter's original justification is now lost: in the 17th century the

[2] Most "small" Vlacq tables did not have differences printed in separate columns, as did later tables.

sine value was proportional to the radius of the circle enclosing the angle. As negative numbers were generally disliked at the time, the radius was chosen to be 10^{10} (10,000,000,000) so that all logarithmic values were increased by 10 and even the smallest trigonometrical logarithm in a table ended up positive. This convention has made calculations on combinations of numbers and trigonometrical functions less intuitive: when mathematical notation progressed from proportional statements to more general formulae, the logarithmic table manuals needed warnings always to subtract 10 from the log-sine and other trigonometrical entries. For some reason Gunter put the wrong radius value 10^8 (instead of 10^{10}) in his introductory text (which generally went unnoticed).

[3] Briggs, H., *Arithmetica Logarithmica, sive Logarithmorum Chiliades Triginta, Pro numeris naturali serie cescentibus ab unitate ad 20,000' et a 90,000 ad 100,000,* London, W. Jones, 1624. From the introduction:

> "Thus, if there is anyone,who wants to complete the gap between the Twentieth and the Ninetieth Chiliads, and on the calculation of these, the value of the work accomplished would itself be considered, to make me sure that it had been carried out in a worthy manner; I will show him how easy it is to add a Chiliad, lest the venture should fall by an ineffective attempt; in the same way that another [Chiliad] be undertaken and completed by someone else.I even have the paper, which has been prepared with this end in sight, and with squares with distinct straight lines, this I can carefully send.And when all the intervening space has been filled up, I will give the work, if it has been done properly, to be printed again."

[4a] Decker, E. de, *Eerste deel van de Nieuwe Telkonst, inhovdende verschevdene manieren van rekenen, waer door seer licht konnen volbracht worden de Geometrische ende Arithmetische Questien,* with a promise for part II: the *Great Table,* Gouda, P. Rammaseyn, 1626.

[4b] Decker, E. de, *Nieuwe Telkonst, inhoudende de logarithmi voor de ghetallen beginnende van 1 tot 10000, ghemaeckt van HENRICO BRIGGIO Professor van de geometrie tot Ocxfort. Mitsgaders de Tafel van Hoeckmaten ende Raecklijnen door het ghebruyck van Logarithmi, de Wortel zijnde van 10000,0000 deelen, gemaeckt van Edmund. Guntero, Professor vande Astronomie tot Londen,*Gouda, P. Rammaseyn, 1626 (published by de Decker together with the *Nieuwe Telkonst I,* again with a promise for part II: the *Great Table* .

[5] Decker, E. de, *Tweede deel van de Nieuwe Telkonst, ofte wonderlicke konstighe tafel inhoudende de Logarithmi, voor de getallen van 1 af tot 100000 toe,* Gouda, P. Rammaseyn, 1627; only one copy known since 1920 (the *Great Table*). (Also available, partly in facsimile, with an introduction by A.J.E.M., Nieuwkoop, Smeur, 1964). De Decker's Preface follows here, translated from Dutch into English:

> Preface to the Art-loving Reader.
>
> Dear Reader, we now bring to light the second part of our New Arithmetic (*Nieuwe Tel-konst*), explained for use in Arithmetic Questions. The reason why here not everything is contained that we have promised before, is that we have decided to publish the use of Logarithms in two different ways. First for those that only need them for Arithmetic: otherwise, for use by lovers of the Arithmetic and Mathematical Arts. We have tried to satisfy the first, and the others we hope to satisfy soon; and because those who are not experienced in the use of Mathematical Arts would not be able to understand the fundaments of the logarithms, we have not decribed here their origin and construction, but only their use–extensively—as many persons only need that.
>
> This Art and new Practice of computing has been invented and described by Ioannes Neperus Lord of Marchistoun in Schotlandt, but he could not work on this table as he died shortly after [his publication]; then Henrico Briggius Professor of Geometry in Oxfoort, in England, has explained and computed with great effort the Logarithms from 1 to 20000 and 90000 to 100000, leaving a gap between 20000 and 90000 which we have tried to fill, with particular great effort, and as my daily profession did not allow me to spend full time, I welcomed the assistence and diligence of the art-loving Adriaen Vlack, who completed successfully most of these required [logarithms] with steady devotion.
>
> This table being such, that one could call it rightly the "Golden Table", given its great benefit and the important works one can accomplish with it.
>
> The importance and the use of logarithms in the full field of mathematics, will soon be published by Adriaen Vlack in Latin, French and Dutch; with a Canon Triangulorum containing the Sines, Tangents and Secants with radius 1,00000,00000, including the differences; Enjoy meanwhile this book, until the other will leave the printing press.

[6] Briggs, H., Vlacq, A., *Arithmetica Logarithmica,* Ed. 2, aucta per A. Vlacq, Gouda, P. Rammaseyn, 1628 (in Latin and French). Following below is the first half of the French "Preface au Lecteur":

> "Il y a environ deux ans; qu'il me vint d'adventure entre les mains l'Arithmetique

Logarithmetique de Monsieur Henry Brigs Professeur en Geometrie à Oxford. Dont ayant veu le Contenu, comme je me trouvay grandement satisfaict de l'Excellence & rarité d'une si belle In- vention, la voyant extremement utile à l'Arithmetique & Geometrie; ainsi d'autrepart en la communiquant avec quelques uns, je recognut bien tost que c'estoit un ouvrage tout incognu en ce Pays Bas, tant envers les plus Doctes Mathematiciens, qu'entre les vulgaires Practiciens. Et en voyant le defaut des Exemplaires, afin que l'art ne demeurast plus caché en ce pays, pensay faire chose aggreable à plusieurs de faire remettre ledit Livre sur la Presse: sans toutesfois estre resolu du commencement en quelle maniere je le devrois plublier. A la fin trouvay bon de l'agencer ainsi qu'il vous est icy presenté: à quoy faire j'ay esté induit per diverses raisons, que je commenceray icy à deduire: Et premierement je diray pourquoy j'ay tasché à suppléer le reste qui defailloit en la Table publiée l'An 1624 par l'Autheur, à savoir, les Logarithmes de tous les nombres depuis 20000 jusques à 90000: Comme aussi pourquoy je les ay abregé de quatre Characteres. Il est vray que le susdit Autheur avoit declaré en sa Preface, qu'il estoit d'advis d'accomplir iceux Logarithmes à l'aide de quelques personages, & (hormis aucune incommodité) de faire alors imprimer de nouveau l'ouvrage tout entier & accompli; de façon que je me susse bien passé de ce labeur, n'eust esté que je me faisois accroire que l'accomplissement de la calculation d'un si grand nombre de Logarithmes, puis aussi l'impression d'un si grand Volume, retardée par les occupations ordinaires de sa Vocation, ou autres quelconques empeschements, qui n'adviennent que trop ordinairement à chasqu'un homme, requerroyent un trop long temps pour satisfaire à mon desir d'avoir lesdites Tables accomplies & d'un user pour ma speculation & contentement particulier. Ioint encore, que l'ouvrage estant tout achevé, je n'estoys nullement asseuré, qu'on auroit moyen d'en avoir assez d'Exemplaires en ce pays, moins encore s'ils feroyent publiez en autre langue que Latine: dont le profit (ce me semble) ne reviendroit qu'à bien peu de personnes. Me voyant doncques assez de loisir, & que les Logarithmes abregez de quatre Characteres, sont beaucoup plus faciles à construire que Henry Brigs ne demonstre en son Chap. 13e. j'ay prins le courage d'entreprendre & achever en assez peu de temps toute l'oeuvre, comme j'ay aussi menée à fin, Dieu aidant, selon le terme que je m'avois proposé. Et ce qui m'a meu de les abreger ainsi de quatre lettres, c'est pource qu'ils sont encores grands assez pour l'usage commun. Et estant plus grands, qu'on ne s'en peut servir sans beaucoup de travail pour des grands nombres, qui sont de plus de dix Characteres, comme il le declare luy mesme en la premiere Edition chap. 14e. & en la presente Chapitre 12 e. apres lequel j'ay adjousté la maniere de trouver facilement Logarithme convenable à chasq'un nombre, qui ne contienne pas outre dix Characteres, & au rebours. Ie confesse que les Logarithmes estant plus grands, qu'on s'en peut servir pour l'une & l'autre intention; mais alors il faudroit, en suivant la maniere d'Henry Brigs, que la Table prenne plus d'espace d'un tiers, & par consequent seroit ce livre plus coustable. Dont, pour servir selon mon pouvoir principalement à ceux de ma Patrie, je l'ay voulu dispenser, me semblant chose non necessaire pour l'usage commun. . . "

[6a] Letter from H. Briggs to J. Pell, Oct. 26, 1629. See [11]: "Letters on Scientific Subjects", published by the Historical Society of Science, 1841, edited by Mr. Halliwell:

> "My desire was to have those Chiliades that are wantinge betwixt 20 and 90 calculated and printed, and I had done them all almost by myselfe, and by some frendes whom my rules had sufficiently informed, and by agreement the busines was conveniently parted amongst us; but I am eased of that charge and care by one Adrian Vlacque, an Hollander, who hathe done all the whole hundred Chiliades and printed them in Latin, Dutche and Frenche, 1000 bookes in these 3 languages, and hathe sould them almost all. But he hathe cutt of 4 of my figures throughout; and hathe left out my Dedication, and to the reader, and two chapters the 12 and 13, in the rest he hathe not varied from me at all."

[6b] Decker, E., de, *Nieuwe Rabat-tafels, waer door sonderlingh licht ende perfect gevonden wort het gereet gelt van eenige somme die te betalen is over eenige Maenden, het Rabat afghetrocken zijnde teghens 8, 9, 10, 11 ofte 12 ten hondert in 't Iaer : mitsg. van Interest op Interest ... : noch is daer by gevoeght de Thiende van Sym. Stevin ...,* Gouda, P. Rammaseyn, 1630.

[6c] Decker, E., de, *Practyck van de groote zee-vaert,* Gouda, P. Rammaseyn, 1631.

[6d] Vlacq, A., *Ephemerides motuum coelestium ad annos vulgares aerea 1633, 1634, 1635, et 1636 in luminarium motibus et zyzygiis ex Tabulis Lansbergianis ab*

Adr. Vlacco, in reliquis planetis ex Tabulis Rudolphinis a Joa. Keplero supputata; cum Instructione super earum usu, Gouda, P. Rammaseyn, 1632.

[7] Briggs, H., and Gellibrand,H., *Trigonometria Britannica,* Gouda, P. Rammaseyn, 1633.

[8] Vlacq, A., *Trigonometria Artificiali,* Gouda, P. Rammaseyn, 1633.

[9] Hutton, C., *Mathematical Tables,* 1785. (4th Ed. in 1804). In his preface, Hutton gives a very interesting introduction with history, construction and usage of logarithms. In the history part, p.38, he cites Richard Norwood's introduction to his *Trigonometrie* of 1631, condemning Vlacq's 1628 publication (without mentioning him by name, however):

> " ... And here I have just occasion to blame the ill dealing of these men [Vlacq and Miller, just having published the 1631 English edition of *Arithmetica Logarithmica*, Part II, both in the matter before mentioned, and in printing a second edition of his [Briggs'] *Arithmetica Logarithmica* in Latin, whilst he lived, against his mind and liking; and brought them over to sell, when the first were unsold; so frustrating those additions which Mr. Briggs intended in his second edition, and moreover leaving out some things that were in the first edition, of special moment: a practice of very ill consequence, and tending to the great disparagement of such as take pains in this kind."

[10] Glaisher, J.W.L., "Notice respecting some new facts in the early history of logarithmic tables", and "Supplementary remarks on some early logarithmic tables", *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science,* XLIV, 4th series, 1872, pp291-303.

[11] Glaisher, J.W.L., "On early logarithmic tables, and their calculators", *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science,* XLV, 4th series, 1873, pp376-382.

[12] Glaisher, J.W.L., "On Errors in Vlacq's Table", *Monthly Notice of the Royal Astronomical Society,* May 1872, pp255-262, extended in June 1873, pp330-345 and pp440-458.

[13] Bierens de Haan, D., *On certain early logarithmic tables,* Rome, 1873, and also *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science,* XLV, 4th series, 1873, pp371-376.

[14] Bierens de Haan, D., *Notice sur des tables logarithmiques hollandaises,* Rome, 1874.

[15] Bierens de Haan, D., "Overbrenging der Briggiaansche logarithmen op het vaderland door Nederlanders", in *Bouwstoffen voor de geschiedenis der wisen natuurkundige wetenschappen in de Nederlanden,*Verhandelingen der Koninklijke Akademie van Wetenschappen, 1874.

[16] Bierens de Haan, D., *Tweede ontwerp eener naamlijst van logarithmentafels, met opgave van den tijd, de plaats en de grootte, alsmede van het aantal decimalen alles zoo verre bekend,* Verhandelingen der Koninklijke Akademie van Wetenschappen, no. 15, 1875.

[17] Braunmühl, A. von, *Vorlesungen über Geschichte der Trigonometrie, 2er Teil,* Paragraph 4 & 5, Leipzig, Teubner, 1903.

[18] Knott, C.G., *Napier Tercenary Memorial Volume,* London, Longmans, 1915. (Many interesting papers on logarithmic tables).

[19] Henderson, J., "Bibliotheca tabularum mathematicarum; being a descriptive catalogue of mathematical tables", *Tracts for computers,* no. 13, Cambridge University Press, 1926.

[20] Kossmann, E.F., *De boekhandel te 's-Gravenhage tot het eind van de 18de eeuw,* Den Haag, 1937, p. 435-443.

[21] Endenburg, P.J.T., "De oudste Nederlandsche logarithmen-tafels en hun makers", *Het boek,* 25 (1938/1939), pp311-320. This paper contains the details of Vlacq's notarial documents, from "Goudse Acten" in the regional archives (Gouda, Streekarchief Hollands Midden).

[22] Haaften, M. van, "De Decker, Vlacq en de oudste logarithmentafels", *De Verzekeringsbode,* 60, 3 & 10 January, 1941.

[23] Fletcher, A., "Early Logarithmic Works", *Nature,* 148, December 13, 1941, p728.

[24] Miller, L., "Milton and Vlacq", *The Papers of the Bibliographical Society of America,* 73, Second Quarter, 1979, pp145-191.

[25] Bruins, E.,M., "On the history of logarithms, Bürgi, Napier, Briggs, De Decker, Vlacq, Huygens", Janus, LXVII, 1980, pp241-259.

[26] Zijden, Th. van der, "Dutch work on logarithmic tables by Adriaen Vlacq", *Proceedings of IM2000,* International Meeting for Slide Rule Collectors, 2000, pp13-19.

[27] Ed.: Campbell-Kelly, M. a.o., *The History of Mathematical Tables,* Chap. 2 (by G. Jagger), Oxford University Press, 2003.